STRUCTURES SUPPORTING LOADS

A spider’s web is a structure made of long, thin strands that must resist pulling forces. The human skull is curved and resists knocks and forces that could otherwise squash the brain. The skull and a spider’s web are just two of the many structures found in nature; they are made from different materials and they have different shapes. People also build structures like bridges and buildings from a range of materials. These structures need to be strong enough to resist forces that try to pull, squash, bend or twist them.

Structures are constructed by people for a particular purpose. For example, buildings enclose spaces principally to protect humans and their belongings from the weather, while bridges enable us to cross openings or gaps created by natural features such as rivers or gorges. Although the design of these structures might be different, the one thing they all have in common is that they must support loads without falling down, breaking, or deforming too much.

Civil engineers are responsible for the structural design of bridges and buildings. The design process involves making decisions about the type of structure to be used, bearing in mind the forces acting on it. Its own weight and the traffic that it carries are just two examples of the forces that a bridge must resist. Similarly, a building must resist its own weight and contents. Loads can also be caused by the environment; they might be due to wind, water, earthquake or changes in temperature. It is necessary for a designer to understand the loads acting on a structure and the reaction forces at the supports (usually the ground). By understanding how the forces are transmitted through a structure to its supports, the designer can decide on the size of the components and the materials to be used. Newton’s laws of motion are fundamental to the design of stable structures.

EQUILIBRIUM — STAYING IN PLACE

The net external force acting on a stationary object must be zero because its acceleration is zero. When this condition is satisfied, the object is said to be in translational equilibrium. For the condition of translational equilibrium to be satisfied in structures such as bridges and buildings, the loads acting on the structure must be balanced by the reaction forces acting at the supports. If translational equilibrium were not satisfied, the structure would accelerate in the direction of the net force.

When the net force acting on a stationary object is zero, the object is said to be in translational equilibrium.

Figure 1: The bridge is supported at both ends and there are three loads acting on it: the person, the dog and the bridge’s own weight. These forces balance each other, so \( F_{\text{net}} = 0 \) and \( P_1 + P_2 + P_3 + N_1 + N_2 = 0 \).
When adding forces to determine the net force, it is often helpful to separate forces into their vertical and horizontal components. (See figure 2.)

### Turning effect of a force

A **torque** is the turning effect of a force.

The turning effect of a force is called a **torque** (τ). A torque can be produced by a force acting at a distance from a point of rotation. When you open a door, turn on a tap or use a spanner to turn a nut, a torque causes rotation. The three variables that influence the magnitude of a torque are:
- the size of the force
- the distance between the force and the point of rotation
- the direction of the force in relation to the point of rotation.

When opening a door, you do not push or pull the handle sideways; the force is most efficient when it is perpendicular to the door. The door is also easier to open when you apply the force further away from the hinge.

![Figure 2: Separating forces into their vertical and horizontal components](image)

#### Figure 2
- \( T_v = T \sin \theta \)
- \( T_h = T \cos \theta \)

#### Figure 3
- (a) A hinged door does not open when you push or pull the handle towards the hinge because there is no turning effect.
- (b) The turning effect of the force is most efficient when the applied force is perpendicular to the door.

#### Figure 4
- (a) When the lever arm (τ) is small, a larger force is needed to open the door.
- (b) When the lever arm is large, a smaller force can be used to open the door.

#### Figure 5
- Calculating torque: \( \tau = r \times F \sin \theta \)

#### Figure 6
- The component of a force that is perpendicular to the door (500 sin θ N) causes a torque and therefore a rotation about the hinge.
- The component of the force parallel to the door (500 cos θ N) does not contribute to the torque about the hinge of the door.
Sample problem 1

The torque needed to open the hinged gate shown in figure 7 (a) to (d) below is 250 N m. In which of the four different cases will the gate open?

**Figure 7:** (a) The force of 100 N is perpendicular to the gate and acts 2 m from the door hinge. (b) The force of 100 N acts 4 m away from the hinge but it is not perpendicular to the gate. (c) The force of 100 N is perpendicular to the gate and acts 4 m from the hinge. (d) The force of 100 N acts 4 m away from the hinge, but it is parallel to the gate.

**SOLUTION**

(a) If we assign clockwise torques as positive, the torque generated is:
\[ \tau = d \times F \]
\[ \tau = 2 \text{ m} \times 100 \text{ N} \]
\[ \tau = 200 \text{ N m} \]

which is less than 250 N m, so the gate will not open.

(b) Using only the component of the force that is perpendicular to the gate, we can find the torque generated:
\[ \tau = dF \sin 45^\circ \]
\[ \tau = 4 \text{ m} \times 100 \text{ N} \]
\[ \tau = 280 \text{ N m} \]

which is greater than 250 N m, so the gate will open.

(c) The torque generated is:
\[ \tau = d \times F \]
\[ \tau = 4 \text{ m} \times 100 \text{ N} \]
\[ \tau = 400 \text{ N m} \]

which is greater than 250 N m, so the gate will open easily.

(d) There is no torque about the hinge and the gate will not open.
Rotational equilibrium

If a structure is in equilibrium, the torque caused by one force must be balanced by the turning effect of one or more other forces. The net torque on the structure must be zero. If the structure was not in rotational equilibrium, it would rotate at an increasing rate.

Engineers must ensure that the torques acting on a structure balance. Balance occurs when the torques that cause a clockwise rotation equal those that cause an anticlockwise rotation. This is something that you might have experienced when playing on a seesaw. To maintain the seesaw in rotational equilibrium, the torque created by the other person must be equal to your torque, or $\Sigma \tau = 0$.

Consider the two people on the seesaw in figure 8. Assume the mass of the seesaw is negligible, and that one person is larger than the other. We know from experience that, to balance the seesaw, the larger person needs to sit closer to the middle. This can also be shown mathematically. The reaction force acting on the seesaw through the support, or fulcrum, is determined from $W_1 + W_2 - N = 0$. However, the seesaw will balance only if the clockwise torques acting on the seesaw are balanced by the anticlockwise torques. Person 2 is tending to turn the seesaw clockwise, while person 1 is tending to turn the seesaw anticlockwise. Taking torques about the fulcrum and assigning clockwise torques as positive and anticlockwise torques as negative, we find:

$$\tau_{\text{net}} = 0$$

$$-d_1 \times W_1 + d_2 \times W_2 = 0$$

$$d_1 = \frac{W_2}{W_1} \times d_2.$$

This equation demonstrates that to maintain rotational equilibrium, the larger person needs to sit closer to the fulcrum, that is $d_1 < d_2$.

Figure 8: To balance the seesaw, the heavier person must sit closer to the turning point at the middle of the seesaw.

All points on a stationary structure are in equilibrium. Therefore, when using rotational equilibrium to determine one or more forces acting on a structure, the point about which torques are calculated will not affect the results. However, the calculations can be simplified by choosing a point through which one or more unknown forces act. This is demonstrated in the following example.
Sample problem 2

Where should person 1 sit to balance the seesaw?

Figure 9

SOLUTION
To satisfy equilibrium, both the sum of the forces acting on the seesaw and the sum of the torques must equal zero.

From $F_{\text{net}} = 0$:
\[800 \text{ N} + 600 \text{ N} - R = 0 \implies R = 1400 \text{ N upwards.}\]

Taking torques about the fulcrum, $\tau_{\text{net}} = 0$:
\[-d_1 \times 800 \text{ N} + 2 \text{ m} \times 600 \text{ N} = 0 \implies d_1 = 1.5 \text{ m}.\]

Alternatively, torques could have been taken about person 1:
\[-d_1 \times 400 \text{ N} + (d_1 + 2) \times 600 \text{ N} = 0 \implies d_1 = 1.5 \text{ m}.\]

Taking torques about person 1 gives the same answer as when torques are taken about the fulcrum.

Try this: take torques about person 2 and see if you get the same answer. Was the calculation as simple as when the torques were taken about the fulcrum?

Investigation 1 — Getting the hang of things

Determine the supporting forces in a simple beam by using a ruler, spring balances and retort stands as shown in figure 10.

1. Before a mass is added to the ruler, do the spring balances measure zero? If not, why not?
2. When you add a mass to the ruler, record its position and the tension in each spring balance. Are the forces in equilibrium? Calculate the torque about one of the supports to determine whether rotational equilibrium is satisfied.
HOLDING THINGS UP

Buildings enclose one or more spaces principally to protect people and their belongings from the weather. In buildings like our houses, the floors support the occupants and the walls hold up the roof and upper floors. In many larger multi-storey buildings, the walls protect the occupants from the weather while a system of columns and beams holds up the floors and roof.

Except for some structures like tents and igloos, the walls of most buildings are vertical. For centuries, stones and bricks have been a convenient means of constructing walls. This is because they are easy to transport and can be easily stacked. Whatever material the walls are built from, they must be able to support their own weight as well as the loads from floors and roofs which push vertically down the wall. The load is resisted by the ground at the base of the wall. Even though the reaction from the ground is distributed over the base of the wall, it is often depicted on diagrams as a single line of force. For a symmetrically loaded wall, column or tower, the reaction force is evenly distributed over the base (see figure 11(a)). It can be represented by a single force acting up through the middle of the base (see figure 11(b)).

Figure 11: For a symmetrically loaded wall, the reaction from the Earth behaves as if it were a concentrated force passing through the middle of the wall.

TOPPLING OVER

Something that topples over is said to be unstable. A vertical structure built with a lean might be at risk of collapsing. The Leaning Tower of Pisa is something that has been at risk of toppling over for some time, but it has not done so! Let’s investigate why.

When the centre of mass of an object lies above its base, the object is stable. The normal reaction balances the weight. When the object is tilted, its weight produces a torque about the base. As long as the centre of mass remains above the base, the torque acts to restore the tilted object to its original position. However, when the centre of mass is no longer above the base, the torque acts to increase the tilting and the object topples over.
Investigation 2 — Balancing

Stand with your back against a wall and with your heels touching the wall. Keeping your legs straight and, without moving your feet, try to bend forwards to touch your toes or ankles. What happens?

Now stand clear of the wall and any furniture or other objects. While keeping your legs straight, bend forwards and try to touch your toes or ankles. Are you able to?

Jo (see figure 14(a)) is not able to maintain his balance. The two forces acting on Jo are his weight, \( W \) (acting through his centre of gravity), and the reaction from the floor, \( R \). Jo's centre of gravity is not over his feet, which are supporting him. Although translational equilibrium is satisfied, that is \( W + R = 0 \), the force \( W \) is not acting along the same line as \( R \). Rotational equilibrium is not satisfied; the pair of forces cause a rotation and consequently Jo loses his balance.

The two forces acting on Pat (figure 14(b)) are her weight, \( W \) (acting through her centre of gravity), and the reaction from the floor, \( R \). When Pat moves her centre of gravity so that it is over her supporting feet, she is able to maintain her balance. As well as satisfying translational equilibrium, that is \( W + R = 0 \), the two forces \( W \) and \( R \) act along the same line. Rotational equilibrium is satisfied and Pat is therefore able to maintain her balance.

Figure 12: When the vertical line passing through the centre of mass is beyond the base, the torque causes the object to topple over.

Figure 13: The Leaning Tower of Pisa has not toppled over. It is stable because the vertical line passing through its centre of mass passes through the base of the tower.

Figure 14: (a) When Jo’s centre of mass is not in line with the normal reaction from the floor, he is unbalanced and he’ll fall over if he tries to touch his toes. (b) When Pat’s centre of mass is in line with the normal reaction from the floor, she is balanced and can touch her toes.
This principle is also demonstrated when a ruler overhangs the edge of a table, (see figure 15). The two forces acting on the ruler are its own weight and the reaction from the table. When the ruler’s centre of mass is located beyond the edge of the table (as in figure 15(b)), it rotates and falls off the table. This is because the centre of mass of the ruler is not vertically above the supporting force from the table.

Figure 15: (a) A ruler is balanced when its centre of mass is over the table. (b) When the centre of mass of the ruler is moved past the edge of the table, rotational equilibrium cannot be satisfied and the ruler falls off the table.

Consider the forces acting on the wall shown in figure 16. When there is no wind, the reaction of the ground, $R_V$, is vertically below the centre of mass of the wall. ($H$ is the force exerted by the wind towards the centre of mass. It is balanced by a force of equal magnitude ($R_H$) applied by the ground.) The net torque on the wall is zero. When the wind blows, it causes a torque that tends to turn the wall about its base. The reaction of the ground, which is now closer to A, balances the torque caused by the wind. The position of the reaction of the ground can be found by taking torques about the centre of the base of the wall. Assuming clockwise torques are positive:

$$\tau_{\text{net}} = X \times R_V - h \times H = 0$$

$$x = \frac{H \times h}{R_V}.$$

However, the ground can stop the wall from toppling only when $x < \frac{d}{2}$. At the point of toppling, the centre of mass of the wall would be vertically above the turning point, A. Values of $x > \frac{d}{2}$ are not possible — the wall will topple.

Figure 16: A wall in the wind will not topple over provided the torque caused by the wind is balanced by the torque from the reaction of the ground.
Walls and towers can be made to resist large overturning forces by increasing their width or mass. Piers and buttresses are another effective way of strengthening walls without increasing the width of the entire wall. Buttresses can often be seen in European cathedrals and in some of the older large churches and cathedrals in Australia. The wide buttresses along church walls serve to prop the main walls against lateral loads. (See figure 17.)

**Figure 17:** The main walls on cathedrals are stabilised by buttresses and flying buttresses. Decorative pinnacles also increase the weight of the walls.

Piers are seen commonly in brick constructions such as fences and the walls of garages and factories. (See figure 18.) When some old buildings were first built, the weight of the wall was increased by adding statues and pinnacles. The increased weight of the wall ensured that the vertical line acting through the centre of mass of the wall passed through the base of the wall.

**BUCKLING**

As well as toppling over, walls, columns and towers are at risk of buckling. Buckling is the bending that occurs when something is compressed. This can be easily seen with a thin piece of plastic material, such as a ruler. (See figure 19.)
Roofs protect the space between the walls of a building. The main purpose of bridges is to carry traffic over openings such as a river or over a road. The loads that bridges and roofs must support usually act vertically down. Arches, domes, beams and trusses are some of the ways used to support the loads over these openings.

Arches, domes and tunnels

The arch is a curved shape that is strong and occurs naturally in rock formations such as in the Island Arch, near Port Campbell in Victoria's southwest (see figure 20). Stone or masonry blocks have also been used to build arches in buildings and bridges. Arches have been used to span the openings in walls such as over windows, doors and walkways. Last century, basalt was used to build short-span arched bridges throughout Victoria. The basalt blocks were transportable, and the stone withstood the forces that occur within an arch.
To understand the behaviour of an arch, consider the heavy chain being held by Pat and Jo in figure 22. Each link in the chain pulls on the links on each side. As well as pulling down on Pat and Jo, the ends of the chain try to move towards each other. To keep the chain in equilibrium, Pat and Jo must pull outwards while also holding the chain up. If the chain is made heavier, or if it is used to support another load, both the vertical force and horizontal force that Pat and Jo must apply are increased. Pat and Jo also find that they must pull harder if they want to make the chain straighter.

An arch can be thought of as an upside-down cable. Instead of experiencing a pull, the components of the arch experience a push. This means that the ends of the arch push down and out on their supports. The supports resist the forces from the arch by pushing inwards as well as upwards. If the arch is heavier, or if it carries other loads, the forces acting on the supports are larger. If the arch is made flatter, the push outwards from its ends is larger. In both cases the horizontal force resisted by the supports is larger. Buttresses are used in many bridges and buildings to resist these horizontal forces.
When supporting a roof, wall or other vertical load, the blocks that make up an arch are pressed together as the load is transmitted through the arch to the supports at each end. Therefore the material used to build an arch must be strong in compression, and the ends of the arch must be prevented from spreading by walls, buttresses or other horizontal restraints. The sloping banks of rivers and gorges are natural buttresses.

**Investigation 3 — What holds it up?**

Support a sheet of A5 paper (nominally 15 cm × 21 cm) between two supports 15 cm apart. Now place a 5¢ piece in the middle of the paper as shown in figure 25(a). The paper sags. Now place the paper between the supports in the shape of an arch as shown in figure 25(b). The paper now supports the 5¢ piece — it will even support a 10¢ piece!

**Figure 23:** When supporting a vertical load the parts of an arch are compressed. The supports react by pushing up and in at each end. The horizontal force in bridges and buildings is often provided by buttresses.

**Figure 24:** In a flatter arch, the horizontal reaction from the supports will be larger.

When supporting a roof, wall or other vertical load, the blocks that make up an arch are pressed together as the load is transmitted through the arch to the supports at each end. Therefore the material used to build an arch must be strong in compression, and the ends of the arch must be prevented from spreading by walls, buttresses or other horizontal restraints. The sloping banks of rivers and gorges are natural buttresses.

**Investigation 3 — What holds it up?**

Support a sheet of A5 paper (nominally 15 cm × 21 cm) between two supports 15 cm apart. Now place a 5¢ piece in the middle of the paper as shown in figure 25(a). The paper sags. Now place the paper between the supports in the shape of an arch as shown in figure 25(b). The paper now supports the 5¢ piece — it will even support a 10¢ piece!
Beams and towers

Horizontal beams are used to transmit the weight of roofs and floors to the supporting walls in houses, garages and sheds. They are also used in bridges to carry the weight of traffic. Flag poles are vertical beams that must primarily resist the horizontal force of the wind. To determine the distribution of forces from a beam to its supports, the principles of translational and rotational equilibrium are once again used.

Sample problem 3

Consider the painter’s plank supported between two trestles in figure 26. The plank behaves as a simple bridge or beam, and the weight of the painter must be transferred through the plank to the two trestles. What happens if the painter moves towards the left? Assume that the weight of the plank is insignificant.

SOLUTION

When the painter is standing in the middle of the plank, her weight is shared evenly between the two trestles. If the structure is stable, the conditions of translational and rotational equilibrium are met.

\[ \begin{align*}
F_{\text{net}} &= 0: W - R_1 - R_2 = 0 \quad [1] \\
\tau &= 0
\end{align*} \]

Assuming clockwise torques are positive, take torques about trestle 2:

\[ L \times R_1 - \frac{L}{2} \times W = 0 \]

\[ R_1 = \frac{W}{2} \]

and by substituting in [1]

\[ R_2 = \frac{W}{2}. \]

Figure 26: When the painter is standing in the middle of the plank, her weight is evenly shared between the two trestles.
As the painter moves towards the left, more of her weight is supported by trestle 1. (See figure 27.)

\[ F_{\text{net}} = 0: W - R_1 - R_2 = 0 \] \[ \text{[2]} \]

Assuming clockwise torques are positive, take torques about trestle 2:

\[ \tau_{\text{net}} = 0: \]

\[ L \times R_1 - b \times W = 0 \]

\[ R_1 = \frac{W \times b}{L}. \]

Substituting in [1]

\[ R_2 = W \left(1 - \frac{b}{L}\right) \]

i.e. as \( b \) increases, \( R_1 \) increases and \( R_2 \) decreases.

Figure 27: The reaction force from trestle 1 increases as the painter moves to the left.

Tension structures

A tensile force is a pull that makes something longer; it creates tension. The cable of a flying fox or a ski tow is in tension. Ropes, wires and cables are used in tension to support lifts in buildings, to stabilise guy poles and masts, and to hold up tents and awnings. (See figure 28.) Cable-supported bridges enable larger openings to be crossed. In cable-supported bridges, the cables must be anchored at each end with intermediate towers to hold them up.

Figure 28: A system of wires in tension is used to support the protective skin over the Sidney Myer Music Bowl in Melbourne.
Sample problem 4

A three-span cable-supported bridge must support a maximum load of 10 t at midspan. The cables make an angle of 30° with the bridge deck. What is the force in each cable? What is the force in each tower which holds up the cables?

SOLUTION
Calculate the tension in each cable.
Vertically:
\[ 10 \ t - 2 \ T \sin 30° = 0 \]
\[ T = 10 \ t. \]
The force from the cables pushes down on each tower.
Vertically:
\[ 2 \ T \sin 30° - P = 0 \]
\[ P = 2 \ T \sin 30° \]
\[ P = 10 \ t. \]
Each tower experiences a force of 10 t.
QUESTIONS

Understanding

1. The two buckets shown in figure 31 are lifted using a stiff rod. How far from the left-hand end should the rod be picked up so that it remains horizontal (assuming the weight of the rod is negligible)?
   (Ans: 0.56 m)

![Figure 31](image)

2. A truck crosses a concrete girder bridge (as shown in figure 32). The bridge spans 20 m and is supported at each end on concrete abutments.
   (a) Describe what happens to the reaction at each abutment as the truck moves across the bridge from left to right.
   (b) The truck weighs 12 tonne. Calculate the reaction at each support when the centre of mass of the truck is 4 m from the right abutment. (Ans: $9.6 \times 10^4$ N)

![Figure 32](image)

3. Rope guys are used to hold up the tent shown in figure 33. A horizontal force of 500 N is needed at each end to hold the tent up.
   (a) What is the size of the force in each guy? (Ans: 1000 N)
   (b) What is the size and direction of the force in the vertical poles at each end? (Ans: 866 N down)
   (c) What will happen if the tent is pitched on soft ground?

![Figure 33](image)
4. The scaffolding shown in figure 34 has been erected on the side of a wall for repairs.

(a) What forces act on the scaffold?
(b) Which parts of the scaffold are in tension? Compression?
(c) If the person weighs 1 kN, what is the size of the force in each member of the scaffold?
(Ans: 1.4 kN)
(d) What happens to the member forces when the person stands closer to the wall?
(Ans: 1.0 kN)

Application

5. The crane in figure 35 weighs 70 tonne and is mounted on rails 6 m apart. The boom weighs an additional 7 tonne.
(a) With the boom in the position shown, calculate the position of the centre of mass of the crane.
(Ans: 1.36 m to right of centre of tower)
(b) Is the crane stable?
(Ans: Yes)
(c) Can the crane lift a 4 tonne load at a radius of 30 m without falling over?
(Ans: Yes)
(d) Assuming the boom to be in the position shown with no load, calculate the size and direction of the wind force that will cause the crane to topple over. (When the wind blows, assume the resultant wind force acts at cabin height.)
(Ans: 35 kN)

6. The truck crane in figure 36 is able to lift a 20 tonne load at a radius of 5 m. If the weight of the truck is evenly distributed, how heavy must the truck be if it is not to tip over? Assume the weight of the truck is uniformly distributed. (Ans: 36 tonnes = 36 kN)
7. The pedestrian bridge spanning the creek in figure 37 weighs 2 kN. Calculate the reaction at each end when the three people are in the positions shown. (Ans: 1.8 kN at each end)

8. If the weight of the chair and skiers shown in figure 38 is 4.0 kN, what is the tension in the cable of the ski lift? (Ans: 14 kN)
SUMMARY

• A structure is in translational equilibrium if the net force acting on it is zero.
• Torque is a measure of the turning effect of a force. Torque is the product of the component of force perpendicular to the line joining the point of rotation and the point of action of the force and the distance from the point of rotation.
• A structure is in rotational equilibrium if the net turning effect of all forces on it is zero.
• A structure is stable if it is in both translational and rotational equilibrium.